

Chasing Chaos With a Magnetic Pendulum

PHY 300 - Junior Physics Laboratory

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1 Introduction

Chaos expresses itself in certain unique ways that can be observed through careful observation and measurement if one searches for these fine signs of chaos amidst the apparent noise and confusion. There are several ways a chaotic system can be differentiated from statistical indeterminacy.

Several simple laboratory experiments have often been used to identify chaotic systems. Some famous examples are the double pendulum, the RLD circuit, pendulums with oscillating pivots and the magnetic pendulum [1]. In each system varying a certain parameter controls the entry into chaos. Slight changes in this parameter take the systems from periodic to chaotic motion. As this control parameter is varied the system shows initial signs of chaos such as period doubling which can be observed using time series plots, phase plane portraits and Poincare sections. Chaos then also has its unique face in these various graphical representations. In this chase after chaos we used a driven magnetic pendulum and mapped its progress from a linear system to a chaotic one while carefully mapping its progress as it displayed clear signs of the onset of chaos.

2 Theoretical Background

There are four forces acting on the forced pendulum with the ring magnet under it. This is the frictional force which results in damping, the sinusoidal driving force from the motor, the repulsive force between the ring magnet and the magnet on the pendulum and gravity. These interacting forces make the nonlinearities in the pendulum more profound and allow for more variable parameters that may be used individually to see the onset of chaos. This means that the approximation given by $\sin(\theta) \approx \theta$ that is used to solve the equation of the simple pendulum Equation 1 breaks down much earlier and our system becomes visibly nonlinear.

$$\frac{\partial^2\theta}{\partial t^2} = -\frac{g}{l}\sin(\theta) \quad (1)$$

The actual system can be modelled by Equation 2 which takes into account all four of the forces at play.

$$\frac{ML^2}{3}\frac{d\omega}{dt} = -\frac{L}{2}Mg\sin(\theta) + T_{driver}\sin(\phi) - \gamma\omega + \frac{|\theta|}{\theta}L\frac{\mu_o}{4\pi}\frac{m_1m_2}{r_\theta^2} \times \cos(|\theta| + \arctan(-|\frac{h_\theta}{L\sin(\theta)}|)) \quad (2)$$

This is the non linear equation that governs our pendulum and a simple solution can not be found to it. However, for the purposes of our experiment, the solution of this equation is not needed. We can make a direct experimental analysis of the system.[3]

3 Experimentation

3.1 Setup

The experiment and procedure was mostly predetermined by the lab manual[2]. Lab view codes for data acquisition were also available. Set up the experiment as shown in Figure 10[2]. This setup allows for flexibility in driver frequency and minimum distance between magnets.

1. The driving force, the AC motor whose speed is controlled simply by a fan dimmer. Care must be taken not to drive the motor in access of 2 cycles per second as it destabilizes the setup. This allows us to control the frequency of the driving force.
2. The Photogate in conjunction with the Smart timer gives the frequency of the driven oscillations. This was read of from the timer display and noted. It was noted that this reading would sometimes change after the dimmer had been set to a certain value and care was taken to repeatedly observe the smart timer to see whether the reading was the same while taking measurements.
3. The flywheel converts the circular motion provided by the motor into simple harmonic motion. The flywheel can also be used to control the amplitude of the forced oscillations by connecting the connecting shaft at different distances from the center. However, in this experiment the connecting rod was fixed at a distance of approx 2 cm from the center.
4. The rails and bearing rod assembly provide for stability and smooth linear motion. They were oiled before taking readings. The rails are movable/adjustable and care must be taken that its sides are parallel to each other; otherwise the bearings and the connecting shaft will face unnecessary friction. This will result in less control over the driven frequency. Oiled and aligned rails and bearing rod assembly gave minimum and constant readings of 1.0, 1.1, and 1.2 whereas badly aligned rails gave minimum readings of 1.5-1.6.
5. The ring magnet is oriented so that it repels the magnet on the pendulum. The distance from the ring magnet to the magnet on the pendulum was controlled by placing the ring magnet on books of different thickness and then measuring the distance with a meter rule.
6. Weights were placed on the base of the retort stand to prevent movement
7. There was enough natural damping in the system that any further damping using magnets was not needed.
8. The attachment of the pendulum to the rotary motion sensor was loose and this was fixed by fixing folded paper into the empty space.
9. The Vernier LabPro took readings (angular displacement) from the Rotary Motion Sensor. These readings were then analyzed using MATLAB.

3.2 Procedure

In our experiment we can control two parameters, namely f , the frequency of the forced oscillations and d , the distance between the ring magnet and the magnet on the tip of the pendulum. The effect of varying both parameters was first analyzed qualitatively by applying different drive frequencies to various orientation and distances. Once a basic idea of the response of the pendulum was gauged by this qualitative analysis a range was determined for d and f . The response of the pendulum without the ring magnet was also noted. Also note that the accuracy with which d was varied is very limited given the method of its variation.

4 Results and Discussion

4.1 Dependence of the Oscillation Period on the Amplitude of Oscillations

The time period of normal pendulums vary with the amplitude of the oscillation which gives rise to non-linearities in the system. For a magnetic pendulum this effect is visible even for small displacements ??.

To observe this effect the pendulum was displaced at different amplitudes of amplitude in the absence and presence of the ring magnet. The amplitude was estimated by using a protractor placed near the pendulum. One such reading where the pendulum was released from an amplitude of about 70 degrees is shown in Figure 1.

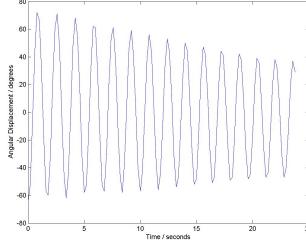


Figure 1: Time Series plot showing slight change in time period in relation to a decrease in amplitude

The results for oscillations without a magnet and oscillations with the repulsive and attractive side towards the pendulum are shown in Figure 2 and compared to published results in Figure 3. Results can be said to agree qualitatively with previous results[1]. This idea is critical to the basis of chaos as it encompasses the essence of feedback into the system. The time period depends on the amplitude. But the amplitude is decreasing because of damping which is proportional to the speed of the pendulum which is related to the time period. This feedback loop coupled with the driven oscillations drives our system into chaos.

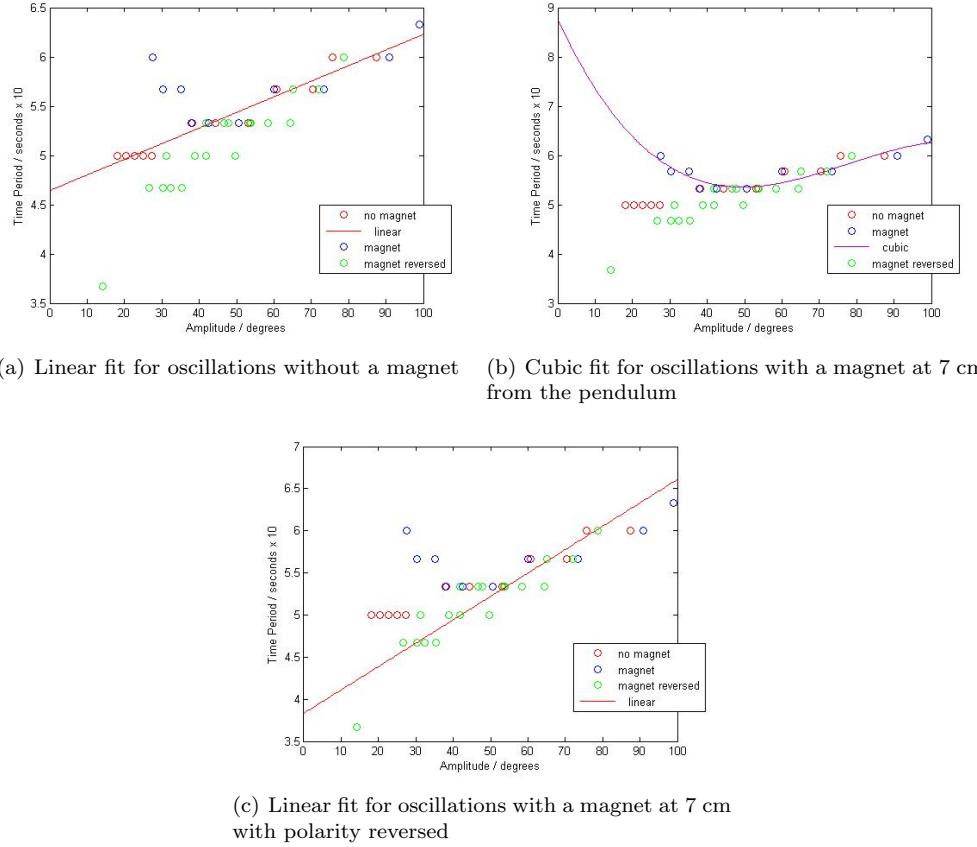


Figure 2: Plots of the time period against amplitude that show how introducing a magnet greatly increases the dependance of the time period of the pendulum on the amplitude of the oscillations.

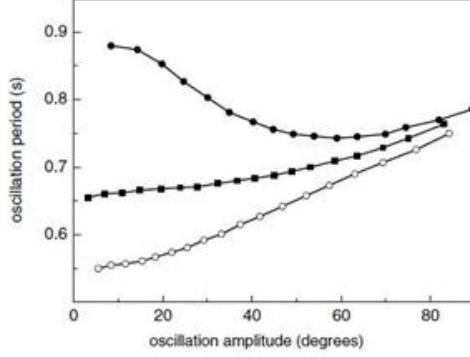


Figure 4. The period of free oscillations versus their amplitude and the potential well of the pendulum: ■—with no current in the coil, ● and ○—two directions of a 0.5 A current.

Figure 3: Results from [1] which show amplitude dependance on the time period, with different orientations of the magnet

4.2 Stepping into Chaos

To observe chaos readings for the following parameters were recorded. D is taken as the minimum distance between the ring magnet and the pendulum and F is the frequency of the forced oscillations of the pivot.

1. For $D = 9$ cm, $F = 1.2, 1.6, 1.8$
2. For $D = 7$ cm, $F = 1.1, 1.6, 1.7, 1.9$
3. For $D = 5$ cm, $F = 1.1$.

4.2.1 Effect of Varying the Drive Frequency

The pendulum in these states had three stable points, one in the center of the pendulum and one to either side. For $d = 7$ and driven frequency of 1.1 Hz we observed periodic motion as shown in Figure 4. As we increased the frequency of the driven oscillations the system became chaotic, Figure 5. From our readings all we can predict is that the transfer into chaos is between 1.2 Hz and 1.6Hz as at 1.6Hz we observed chaotic motion. While the time series plot may look like noise or simply indeterminate results the phase plane plot clearly shows the sign of chaos as the oscillations are attracted to rotate around the three fixed points.

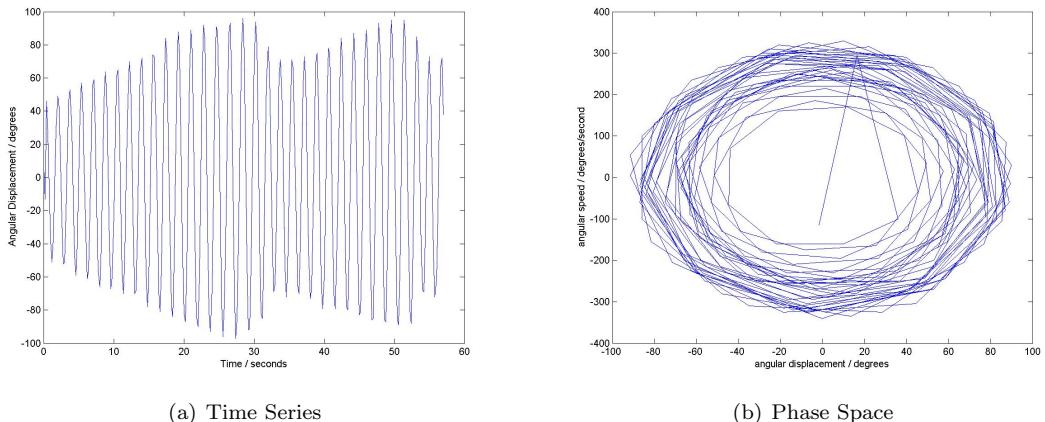


Figure 4: $D = 7\text{cm}$ and $F = 1.2 \text{ Hz}$, time series and associated phase portrait showing periodic motion

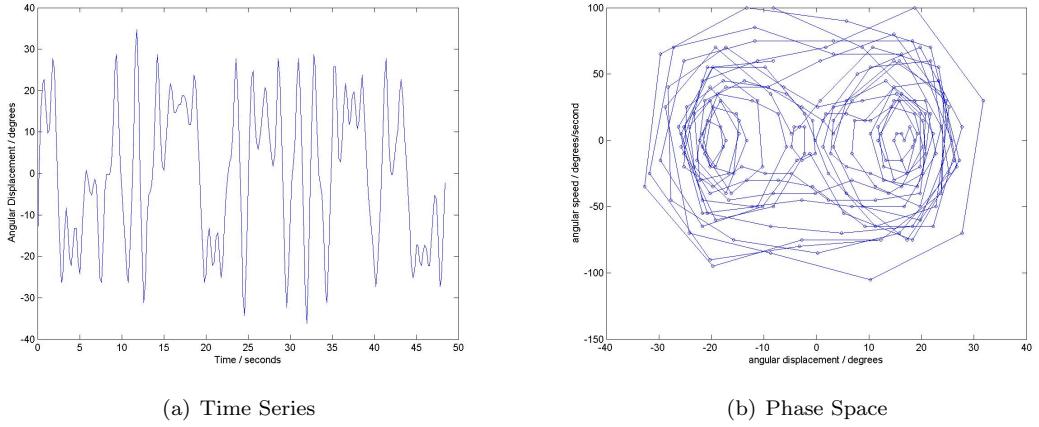


Figure 5: $D = 7\text{cm}$ and $F = 1.6 \text{ Hz}$, time series and associated phase portrait showing chaotic motion

Further increasing the drive speed resulted in a greater splitting between the three fixed points. It could be said that the system became more chaotic. Refer to Figure 6 and Figure 7.

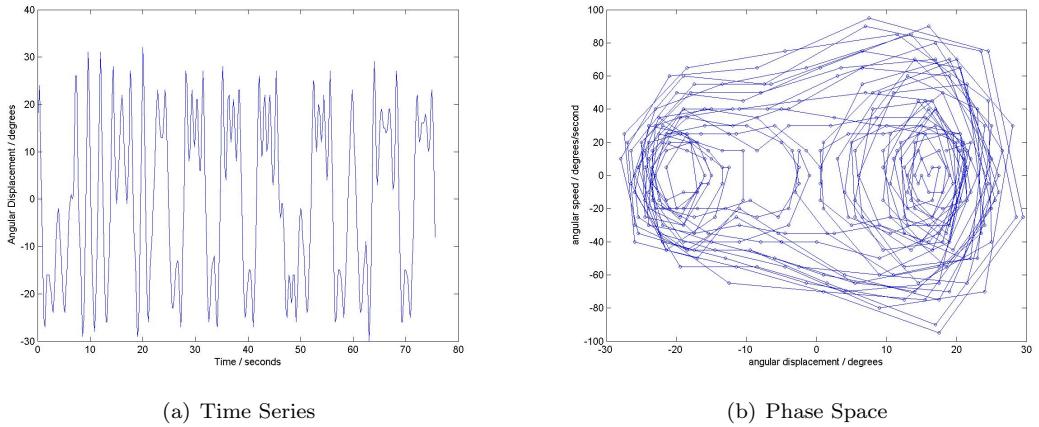
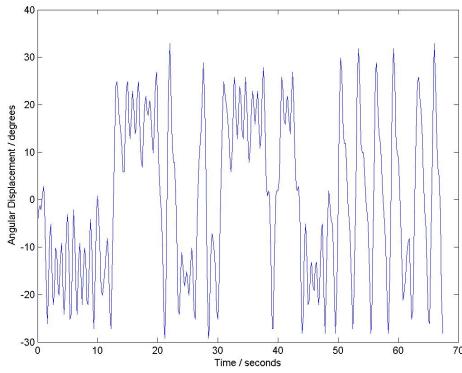


Figure 6: $D = 7\text{cm}$ and $F = 1.7 \text{ Hz}$, time series and associated phase portrait showing chaotic motion

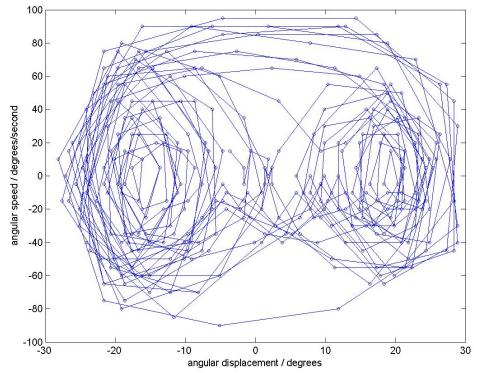
4.2.2 Effect of Varying the distance between the Pendulum and the Ring Magnet

The distance was also varied in our observations. Given past research, such systems are very sensitive to this distance. The system can be seen to enter chaos by simply bringing the magnet closer. While maintaining the same drive speed the magnet should be brought closer until the onset of chaos. This was not done in our experiment. Results for $d = 9\text{cm}$ and $d=7\text{cm}$ while $f = 1.6\text{Hz}$ is shown in Figure 8.

When D was further reduced to 5cm the pendulum could not overcome the repulsive forces of the ring magnet and began to oscillate around one of the exterior fixed points. An example with only positive angular displacements is shown in Figure 9.

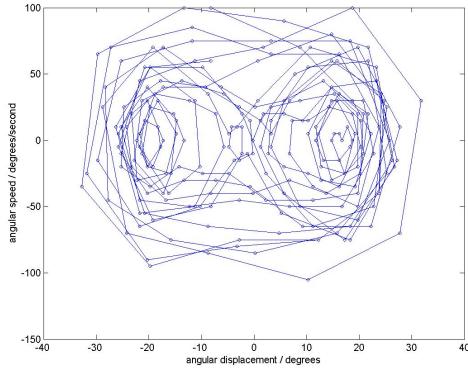


(a) Time Series

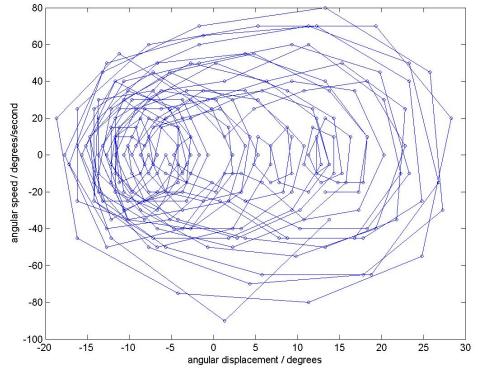


(b) Phase Space

Figure 7: $D = 7\text{cm}$ and $F = 1.9 \text{ Hz}$, time series and associated phase portrait showing chaotic motion

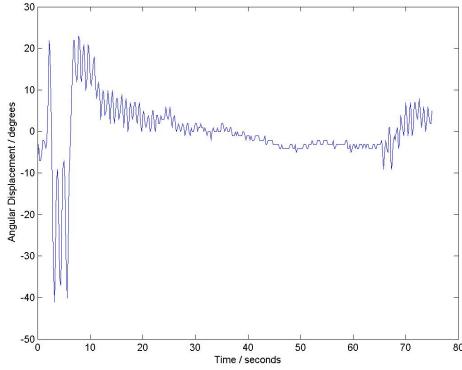


(a) $D = 7 \text{ cm}$

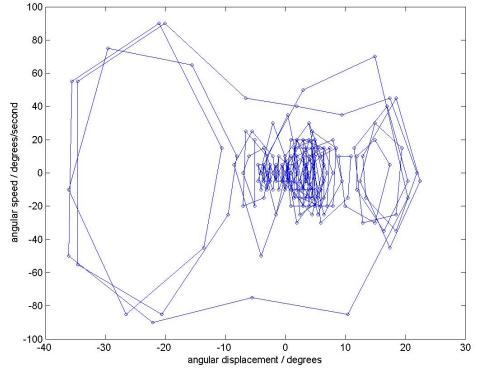


(b) $D = 9 \text{ cm}$

Figure 8: Phase plane plots for driven frequency of 1.6 Hz and different D .



(a) Time Series



(b) Phase Space

Figure 9: For $D = 5 \text{ cm}$ the the degree displacements are only positive, phase plane and times series plot.

4.3 Poincare Sections

Poincare Sections observe a slice of the system after some given time T. If T=time period of the harmonic system then the poincare section will always observe the pendulum in the same position and all points will be close to each other. For bifurcated systems there will be two, four, eight groups of points given the level of bifurcation. For chaotic systems points will be found all over the place.

4.4 Fourier Plots

Fourier transforms, take the time series and extracts the dominant frequencies and plots the frequency spectrum. For harmonic motion there will be one peak. If the system bifurcates one will observe a second peak emerge. For chaotic systems there will be numerous peaks everywhere.

5 Conclusion

The most important conclusion from this experiment is that one should analyze results, while performing an experiment so that missing/ incomplete readings may be easily taken for a better more wholesome analysis. Secondly, chaos is very sensitive to parameters on the boundaries. It is this sudden jump that can be taken as a sign of chaos. The system should have been better analyzed to find this boundary value rather than a general overall picture of chaotic and periodic responses at various parameter values. Chaos, being non-linear should be also sampled at non-linear intervals with larger readings taken near sensitive areas. The magnetic pendulum followed simple harmonic motion at low F and high D but went chaotic as F was increased or D reduced. Both of the variable parameters, the drive frequency and separation between magnets were varied and the responses of the system noted. Chaos and harmonic motion was plotted using only time series, phase plots. The dependance on the time period was shown to vary with the amplitude and the presence of several mean positions demonstrated by reducing D.

References

- [1] A. Siahmakoun, V. A. French, J. Patterson, Nonlinear dynamics of a sinusoidally driven pendulum in a repulsive magnetic field”, Am. J. Phys. 65 (5), May 1997.
- [2] Junaid Alam, M. Sabieh Anwar, ”Magnetic Pendulum”, <http://physlab.lums.edu.pk/>
- [3] Abdullah Khalid, ”The Magnetic Pendulum”, Unpublished, October 2010.

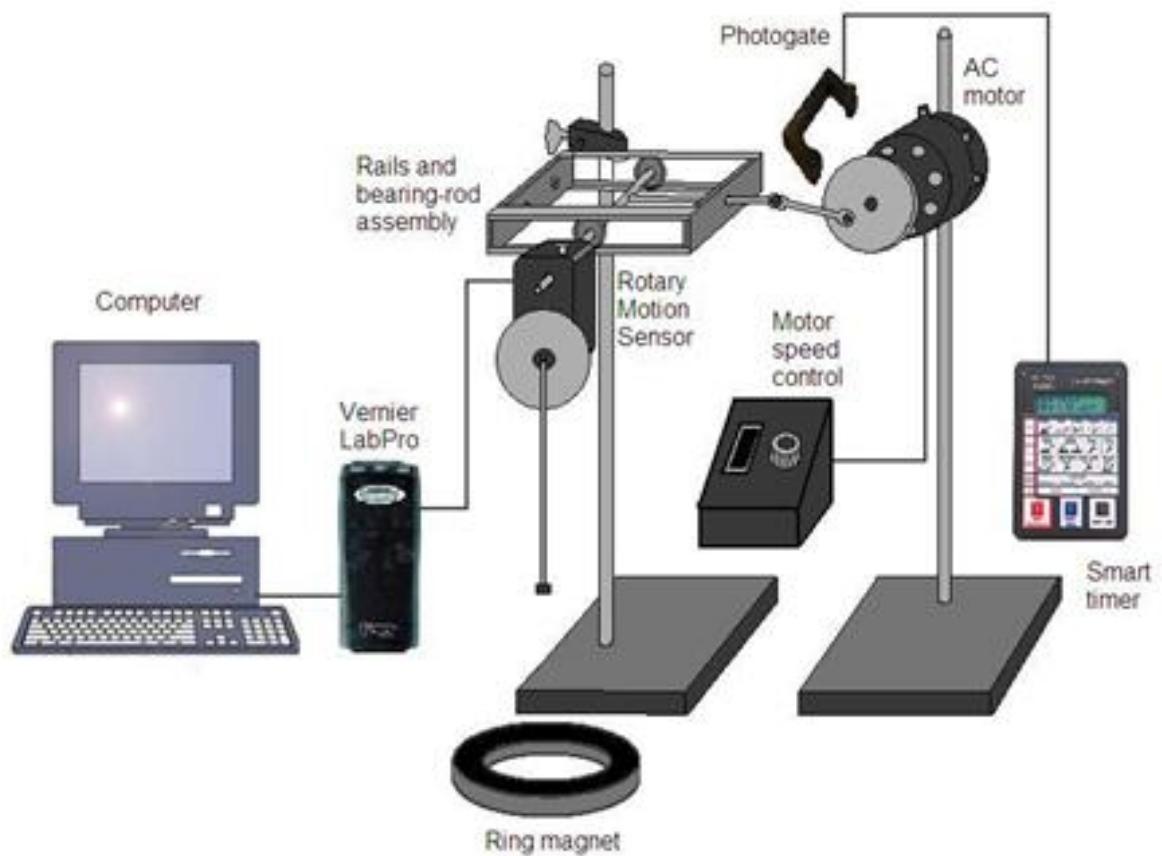


Figure 10: Experimental Setup