

Reading

- Calculus notes at Star Photocopier.

Curve sketching and plotting:

In this problem set you are asked to sketch various functions. When you are asked to *sketch* a function *do not* plot the function by plugging in values of x and computing the corresponding y values. If you are unclear about the difference between sketching and plotting, please talk to us before attempting these problems.

1. Sketch the function $f(x) = 1/x^2$. What is
 - (a) $\lim_{x \rightarrow \infty} f(x)$?
 - (b) $\lim_{x \rightarrow 0} f(x)$?
 - (c) $f(0)$?
2. Sktech the function

$$f(x) = \begin{cases} \sqrt{x-4} & \text{if } x > 4 \\ 8-2x & \text{if } x < 4 \end{cases}$$

Does the $\lim_{x \rightarrow 4} f(x)$ exist? If so, what is it? Can you compute $f(4)$?

3. Estimate

$$\lim_{t \rightarrow 0} \frac{\sqrt{t^2 + 9} - 3}{t^2}$$

4. Sktech the function $f(x) = 1/(x^2 + 1)$. Compute the equation of the tangent at $x = 2$. Draw this tangent line on the graph of $f(x)$. Now on the same graph sketch $\frac{df}{dx}$.
5. Sketch the graph of the following function and use it to determine the values of a for which $\lim_{x \rightarrow a} f(x)$ exists.

$$f(x) = \begin{cases} 2-x & \text{if } x < -1 \\ x & \text{if } -1 \leq x < 1 \\ (x-1)^2 & \text{if } x \geq 1 \end{cases}$$

6. Give an example to disprove the following statement: If $f(x) < g(x)$ for all x , then $\lim_{x \rightarrow a} f(x) < \lim_{x \rightarrow a} g(x)$.
7. Give an example where $\lim_{x \rightarrow a} f(x^2)$ exists but $\lim_{x \rightarrow a} f(x)$ does not.