3.9

$m(t) = (M + m - \frac{dm}{dt} t)$, Then

$F = m(t) \frac{dv}{dt} = (M + m - \frac{dm}{dt} t) \frac{dv}{dt}$, and

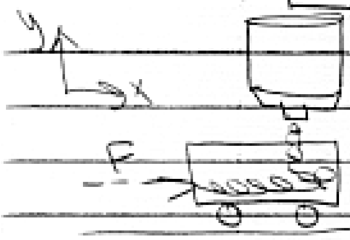
$v = \int \frac{F\,dt}{M + m - \frac{dm}{dt} t}$. From here

$v(t) = \frac{-F}{dm/dt} \ln \left(\frac{m + M - \frac{dm}{dt}}{m+M} \right)$, and

$v_f = \frac{F}{dm/dt} \ln \left(\frac{m+M}{M} \right)$

PS5-6

4. $\boxed{\text{K+K 3.10}}$



Given: An empty freight car of mass M starts from rest under an applied force F. Sand runs in at a steady rate b. ($\frac{kg}{s}$)

Goal: find the speed when a mass m of sand has been transferred.

Strategy: Use Impulse = change in momentum
(in horizontal direction)

Let t be the time to transfer mass m of sand.

$$\Rightarrow \quad bt = m \quad \text{or} \quad \boxed{t = \frac{m}{b}}$$

Impulse during this time is (along x axis)

$$I = \int_0^t F\,dt' = F\int_0^t dt' = Ft$$ since F is constant and is only horizontal force (no friction!)

$$\Rightarrow I = Ft = P(t) - P(0) = (M+m)v_f - 0$$

$$\Rightarrow \boxed{v_f = \frac{I}{M+m} = \left(\frac{m}{M+m}\right)\frac{F}{b}}$$

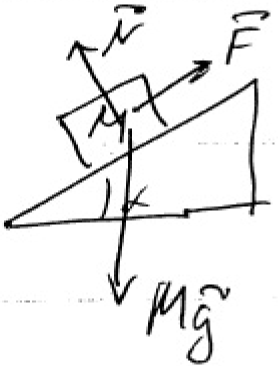
system is car plus sand transferred in time t

Check: $M = 500$ kg, $b = 20$ kg/s, $t = 10$ s $\Rightarrow m = 200$ kg, $F = 100$ N $\Rightarrow v_f = 1.4$ m/s ✓

- units ✓
- $F = 0 \Rightarrow v_f = 0$ ✓
- $b \to \infty \Rightarrow v_f \to 0$ (dump a massive amount of sand, it stops)
- $b \to 0 \Rightarrow v_f \to \infty$ (just continues to accelerate)

3.13

During τ the rope moves by $\Delta l = v\tau$. This is the average ~~number of~~ distance between two skiers. Then the number of them is $N = \frac{l}{\Delta l}$, and their total mass $M = \frac{ml}{v\tau}$. The acceleration is zero:



$\vec{N} + \vec{F} + M\vec{g} = 0$. Project this on the direction of the slope:

$F = Mg\sin\alpha = \frac{ml}{v\tau} g \sin\alpha = 3128\ N$

6 (3.16) Shooting out of the fire hydrant

By Newton's second law, the reaction force is equal to:

$$F = \frac{dP_{water}}{dt} = \frac{d(m_{water}V_0)}{dt} = V_0\frac{d(\rho_{water}V)}{dt} = \rho_{water}SV_0^2. \tag{17}$$

In the last equality we used that in time dt the volume of the shot out water would be $V = SdL$, with $dL = V_0dt$.

Problem 3.18

A raindrop of initial mass M_0 starts falling from rest under the influence of gravity. Assume that the drop gains mass from the cloud at a rate proportional to the product of its instantaneous mass and its instantaneous velocity:

$\frac{dM}{dt} = kMV$, where k is a constant.

Show that the speed of the drop eventually becomes effectively constant, and give an expression for the terminal speed. Neglect air resistance.

Solution:

According to equation (3.11),

$$\vec{F} = \frac{d\vec{P}}{dt},$$

where $\vec{F} = M\vec{g}$, and $\vec{P} = M\vec{V}$

write the equation in its scalar form, one obtains:

$$Mg = \frac{d(MV)}{dt} = M\frac{dV}{dt} + V\frac{dM}{dt}$$

since $\frac{dM}{dt} = kMV$, we see that:

$$Mg = M\frac{dV}{dt} + kMV^2$$

that is: $kV^2 + \frac{dV}{dt} = g$

Then we can see, as the drop falls, it's velocity increases due to gravity. However, there is a limit for this velocity because as the drop speeds up, its acceleration decreases rapidly and will become negligible as the time goes to infinity. So the speed of the drop will eventually become constant.

To compute the terminal constant speed, we could set $\frac{dV}{dt} = 0$

Then we have: $kV_t^2 = g$

Or

$$V_t = \sqrt{g/k}$$

Problem 4.2

(3.20) The rocket moves in a gravitational uniform field by ejecting exhaust with a constant speed u. The rate of expelling mass is $\frac{dm}{dt} = \gamma m$ (apparently, $\gamma < 0$). The rocket is retarded by an air-resistance with a force $m\beta v$. Find the velocity of the rocket as a function of time.

The rocket equation is $F_{external} = m\frac{dv}{dt} + u\frac{dm}{dt}$, or $\cancel{m}\frac{dv}{dt} = -\cancel{m}g - \cancel{m}\beta v - u\gamma\cancel{m}$. The mass cancels

$\frac{dv}{dt} + \beta v + g + u\gamma = 0$. The solution is

$v(t) = Ce^{-\beta t} - \frac{(g+u\gamma)}{\beta}$, where the constant C can be found from the condition that $v(0) = 0$.

$v(0) = C - \frac{(g+u\gamma)}{\beta} = 0 \Rightarrow C = \frac{g+u\gamma}{\beta}$, and

$v(t) = \frac{g+u\gamma}{\beta}(e^{-\beta t} - 1)$