

Ch 4

①

$$4.2 \quad F_f = \mu N = b x M g$$

Work done by F_f in moving a distance x
is given by $\int_0^x F_f dx$

$$= b M g \int_0^x x dx$$

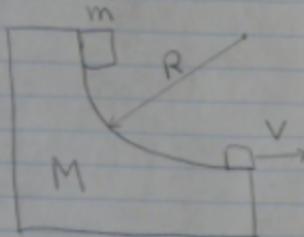
$$= \frac{b M g x^2}{2}$$

P.E. in spring = $\frac{1}{2} K x^2$

$$\frac{1}{2}(K + b M g)x^2 = \frac{1}{2} M V_0^2$$

$$x^2 = \frac{M V_0^2}{K + b M g}$$

4.4



$$mgR = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$$

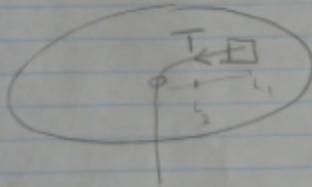
$$mv = MV \implies V = \frac{m}{M}v$$

$$mgR = \frac{1}{2}mv^2 + \frac{1}{2}M\frac{m^2}{M^2}V^2$$

$$gR = \frac{v^2}{2} \left(1 + \frac{m}{M} \right)$$

$$v^2 = 2gR \left(1 + \frac{m}{M} \right)^{-1}$$

4.5



$$V = rw$$

$$\Delta K.E. = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \frac{1}{2}m \left[l_2^2 w_2^2 - l_1^2 w_1^2 \right]$$

$$\ddot{\vec{r}} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$

$$\ddot{\vec{r}} = -mrw^2\hat{r} \quad d\vec{r} = dr\hat{r}$$

$$\int_{l_1}^{l_2} \vec{T} \cdot d\vec{r} = \int_{l_1}^{l_2} -mrw^2 \quad ,$$

(2)

$$2\dot{r}\dot{\theta} + r\ddot{\theta} = 0$$

$$2\dot{r}\omega + r\dot{\omega} = 0$$

$$\int_{t_1}^{t_2} 2 \frac{\dot{r}}{r} dt = \int_{t_1}^{t_2} -\frac{\dot{\omega}}{\omega} dt$$

$$\int_{l_1}^{l_2} \frac{2 dr'}{r'} = - \int_{\omega_1}^{\omega_2} \frac{d\omega'}{\omega'}$$

$$\left(\frac{l_2}{l_1}\right)^2 = \left(\frac{\omega_1}{\omega_2}\right)^2$$

$$2 \ln \frac{r}{l_1} = - \ln \frac{\omega_2}{\omega_1} \Rightarrow \left(\frac{r}{l_1}\right)^2 = \frac{\omega_1}{\omega_2}$$

$$\omega = \frac{\omega_1 l_1^2}{r^2}$$

$$\int_{l_1}^{l_2} \vec{T} \cdot d\vec{r} = - \int_{l_1}^{l_2} m r \omega_1^2 \frac{l_1^4}{r^4} dr = - m \omega_1^2 l_1^4 \int_{l_1}^{l_2} \frac{1}{r^3} dr$$

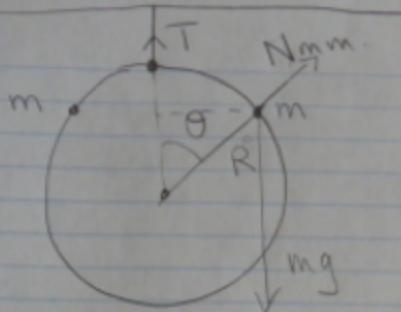
$$= \frac{m \omega_1^2 l_1^4}{2} \left[\frac{1}{r^2} \right]_{l_1}^{l_2}$$

$$= \frac{m \omega_1^2 l_1^4}{2} \left[\frac{1}{l_2^2} - \frac{1}{l_1^2} \right]$$

$$= \frac{m \omega_1^2 l_1^2}{2} \left(\frac{l_1}{l_2} \right)^4 - \frac{m \omega_1^2 l_1^2}{2}$$

$$= \frac{m \omega_2^2 l_2^2}{2} - \frac{m \omega_1^2 l_1^2}{2}$$

4.7



$$N - mg \cos \theta = m a_r = -m R \dot{\theta}^2 = -\frac{mv^2}{R}$$

$$\frac{1}{2}mv^2 = mgR(1-\cos\theta)$$

F_N does no work since L motion

$$N - mg \cos \theta = -\frac{mv^2}{R} \quad (\dagger)$$

$$mgR(1-\cos\theta) = \frac{1}{2}mv^2 \quad (\ddagger)$$

$$\dagger \Rightarrow N = mg \cos \theta - \frac{mv^2}{R} \quad (\ddagger \ddagger)$$

$$\ddagger \Rightarrow \frac{mv^2}{R} = 2mg(1-\cos\theta)$$

Plug into $\ddagger \ddagger$

$$N = mg \cos \theta - 2mg(1-\cos\theta)$$

$$N = 3mg \cos \theta - 2mg$$

(3)

Net vertical force on ring

$$F = T - Mg - 2N \cos\theta$$

$$= T - Mg - 2(3mg \cos\theta - 2mg) \cos\theta$$

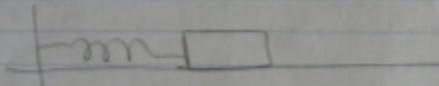
$$= T - Mg - 6mg \cos^2\theta + 4mg \cos\theta$$

Net force $\boxed{F = 0}$ \Rightarrow ring about to lift up
 $T = 0$

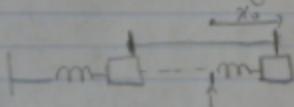
$$Mg = -6mg \cos^2\theta + 4mg \cos\theta$$

Solve for θ .

.8



a) Work done in 1 cycle of oscillation if x_0



equilibrium

$$\frac{1}{2}K(x_0 - \Delta x)^2 = \frac{1}{2}Kx_0^2 - 4fx_0$$

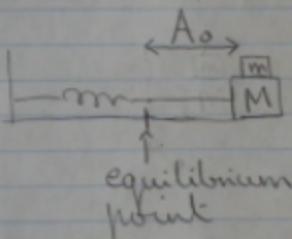
$$\frac{1}{2}Kx_0^2 - Kx_0\Delta x = \frac{1}{2}Kx_0^2 - 4fx_0$$

$$\Rightarrow -Kx_0 \Delta x = -4f x_0$$

$$\Rightarrow \Delta x = -\frac{4f}{K}$$

(b) Since $\Delta x = -\frac{4f}{K}$

$$\# \text{ of oscillations} = \frac{x_0}{\Delta x} = \frac{x_0 K}{4f}$$



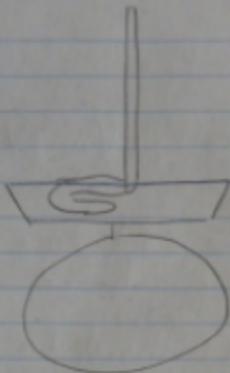
a) $T = 2\pi \sqrt{\frac{M+m}{K}}$

b) amplitude = A_0 .

c) $E = \frac{1}{2} K A_0^2$. No change

(4)

4.11



Two components to the reading.

- 1) weight of chain of length x
- 2) "impact" of chain
- 1) weight = $Mg \frac{x}{L}$

2) When x has fallen onto scale

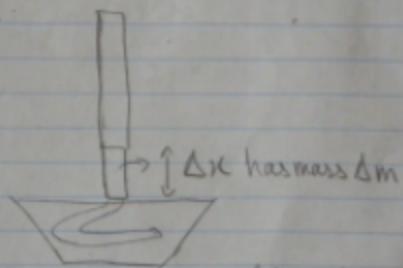
In Δt seconds, a further Δx will fall onto scale.

This Δx length has speed $\frac{1}{2}mv^2 = mgx$

$$\Delta x = \sqrt{2gx} \Delta t$$

$$v = \sqrt{2gx}$$

$$\Rightarrow \Delta m = \Delta x \frac{M}{L} = \frac{M}{L} \sqrt{2gx} \Delta t$$



$$V_f = 0 \Rightarrow P_f = 0$$

$$V_i = \sqrt{2gx} \Rightarrow P_i = (\Delta m) V_i = \frac{M}{L} (\sqrt{2gx})^2 \Delta t$$

$$F = \frac{P_f - P_i}{\Delta t} = -\frac{M}{L} 2gx$$

By Newton's Third Law force on
scale = $+\frac{M}{L} 2gx$.

$$\text{Net force} = 2Mg \frac{x}{L} + Mg \frac{x}{L}$$

$$= 3Mg \frac{x}{L}$$

(5)

4.13

$$U = \epsilon \left[\left(\frac{r_0}{r} \right)^{12} - 2 \left(\frac{r_0}{r} \right)^6 \right]$$

a) $\frac{dU}{dr} = \epsilon \left[r_0^{12} (-12r^{-13}) + r_0^6 (12r^{-7}) \right] = 0$

$$-r_0^6 r^{-6} + 1 = 0$$

$$\Rightarrow r = r_0$$

$$U(r_0) = \epsilon \left[1 - 2 \right] = -\epsilon$$

b) To find frequency of oscillation

$$U(r) = U(r_0) + (r-r_0) \frac{dU}{dr} \Big|_{r_0} + \frac{1}{2} (r-r_0)^2 \frac{d^2 U}{dr^2} \Big|_{r_0}$$

$$\Rightarrow K = U''(r_0)$$

$$\Rightarrow \omega = \sqrt{\frac{U''(r_0)}{m}} \quad \textcircled{*}$$

Find $U''(r_0)$ and plug in $\textcircled{*}$

4.14

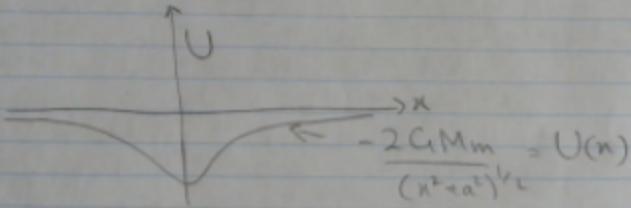
$$a) U(x) = \frac{-2GMm}{(x^2+a^2)^{1/2}}$$

$$b) \Delta P.E. = -\frac{2GMm}{a\sqrt{10}} + \frac{2GMm}{a}$$

$$= \frac{2GMm}{a} \left[1 - \frac{2}{\sqrt{10}} \right] = \text{gain in K.E.}$$

$$\frac{1}{2}mv_f^2 = \frac{2GMm}{a} \left[1 - \frac{2}{\sqrt{10}} \right] + \frac{1}{2}mv_0^2$$

c)



$$U(x) = -2GMm (x^2+a^2)^{-1/2}$$

$$U'(x) = GMm (x^2+a^2)^{-3/2} 2x$$

$$U''(x) = GMm (x^2+a^2)^{-5/2} 2 - \frac{3}{2} GMm (x^2+a^2)^{-7/2}$$

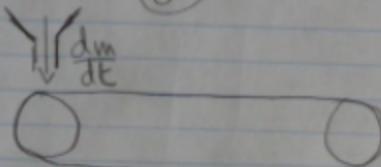
$$U''(0) = \frac{2GMm}{a^3}$$

See 4.13 for details

$$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{2GMm/a^3}{m}}$$

(6)

4.20 a)



$$\frac{dm}{dt} = b$$

In a time Δt , $b\Delta t$ mass falls onto belt

$$P_i = 0$$

$$P_f = Vb\Delta t$$

$$F\Delta t = Vb\Delta t \Rightarrow F = Vb$$

$$\text{Work done} = F\Delta x = VbV\Delta t$$

$$\text{Power} = \frac{\text{Work done}}{\Delta t} = V^2b$$

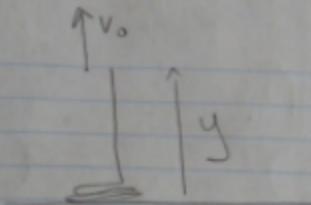
$$\begin{aligned} b) \Delta \text{K.E.} &= \frac{1}{2}(\Delta m)V^2 \\ &= \frac{1}{2}(b\Delta t)V^2 \end{aligned}$$

$$\frac{\Delta \text{K.E.}}{\Delta E} = \frac{1}{2}bV^2 \quad \textcircled{*}$$

Difference in Power and $\textcircled{*}$ is because of friction which is nonconservative

4.21

a)



At time t , height = y

$$\text{At time } t + \Delta t, \text{ height} = y + \Delta y = y + v_0 \Delta t$$

At time t ,

$$P(t) = \lambda y v_0$$

At time $t + \Delta t$

$$\begin{aligned} P(t + \Delta t) &= \lambda y v_0 + \lambda(\Delta y) v_0 \\ &= \lambda v_0 (y + v_0 \Delta t) \end{aligned}$$

$$\Delta P = P(t + \Delta t) - P(t) = \lambda v_0^2 \Delta t$$

$$F = \frac{\Delta P}{\Delta t} = \lambda v_0^2$$

$$\text{Net force} = \lambda v_0^2 + \lambda y g$$

$$(b) P = FV = \lambda v_0^3 + \lambda y g v_0$$

At time t

$$\text{K.E.} = \frac{1}{2} \lambda y v_0^2$$

$$\text{P.E.} = \lambda y g \frac{y}{2} \xleftarrow{\text{center of mass}}$$

At time $t + \Delta t$

$$\text{K.E.} = \frac{1}{2} \lambda y v_0^2 + \frac{1}{2} \lambda (v_0 \Delta t) v_0^2$$

$$\text{P.E.} = \lambda (y + v_0 \Delta t) g \left(\frac{y + v_0 \Delta t}{2} \right)$$

$$\frac{\Delta (\text{K.E.} + \text{P.E.})}{\Delta t} = \frac{1}{2} \lambda v_0^3 \Delta t + \lambda y g v_0 \Delta t = \frac{1}{2} \lambda v_0^3 + \lambda g y v_0$$