University of Engineering and Technology Lahore
Department of Electrical Engineering

MA-100 **Problem Set #4** November 4, 2011

Due: Tuesday, November 15.

Reading

- Calculus lecture notes (from last week).
- Thomas and Finney, section 6.1 (uploaded on `http://www.computerscience.pk`).

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1. Consider the function
$$f(x) = \begin{cases} x & \text{if } x \leq 0 \\ 2 & \text{if } x > 0 \end{cases}$$
Consider the function $g(x) = \int_{-1}^{x} f$. Is the function g differentiable? If so what is the derivative? Does this violate the fundamental theorem of calculus?

2. Consider the function
$$f(x) = \int_{1}^{\cos x} t^2 dt$$
   (a) Compute the integral to show that $f(x) = \dfrac{\cos^3 x}{3} - \dfrac{\cos 1}{3}$.
   (b) Compute $\dfrac{df}{dx}$.
   (c) Now define $g(x) = \int_1^x t^2 dt$. Find a function $h(x)$ such that you can write $f(x) = g \circ h(x)$. Now use the chain rule and the fundamental theorem of calculus to find $Df(x)$.

3. We can generalize the result of the previous problem. In this question g is some function defined on $\mathbb{R}$.
   (a) Define $f(x) = \int_0^{\alpha(x)} g(t)dt$. Compute $Df(y)$.
   (b) Define $f(x) = \int_{\beta(x)}^{\alpha(x)} g$. Compute $Df(z)$.

4. Compute $\int_b^a \tan(x)dx$ using the substitution theorem. You are supposed to use both versions of the substitution theorem: first do it by the method you studied in F.Sc./A-levels and then later on use the substitution theorem as taught in class.

5. Exercise 6.1 (Thomas's Calculus). 1, 2, 5, 10, 12.