Solution: Problem Set 5
Calculus 1

November 25, 2011

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The cross-sections are circular disks with radii given by

$$y = 1 - \frac{x}{2}.$$

Hence the volume is given by

$$\begin{aligned} V &= \int_0^2 \pi \left(1 - \frac{x}{2}\right)^2 dx \\ &= \frac{2\pi}{3}. \end{aligned}$$

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In this case the cross-section is again a circular disk with radius

$$\sqrt{2} - \sec x \tan x.$$

To determine the limits of integration we need to find out the value of x for which $\sec x \tan x$ becomes equal to $\sqrt{2}$. Doing some algebra we can see that

$$\begin{aligned} \sec x \tan x &= \sqrt{2} \\ \frac{\sin x}{\cos^2 x} &= \sqrt{2} \\ \sin x &= \sqrt{2}(1 - \sin^2 x) \\ \sqrt{2}\sin^2 x + \sin x - \sqrt{2} &= 0. \end{aligned}$$

Solving the above quadratic equation for $\sin x$ we get

$$\sin x = \frac{1}{\sqrt{2}} \text{ OR } \sin x = -\sqrt{2}.$$

Discarding the second root (which is not real), we get

$$x = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}.$$

Hence the volume is given by

$$\begin{aligned} V &= \int_0^{\pi/4} \pi \left(\sqrt{2} - \sec x \tan x\right)^2 dx \\ &= \pi \left(\frac{\pi}{2} + 2\sqrt{2} - \frac{11}{3}\right). \end{aligned}$$

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The cross-section is a washer with outer radius 1 and inner radius $\sqrt{\cos x}$. Hence the cross-sectional area is given by

$$A(x) = \pi(1^2 - (\sqrt{\cos x})^2) = \pi(1 - \cos x).$$

The volume can now be calculated as

$$\begin{aligned} V &= \int_{-\pi/2}^{\pi/2} \pi \left(1 - \cos x\right) dx \\ &= \pi^2 - 2\pi. \end{aligned}$$

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In this question as well, the cross-section is a washer with outer radius $4 - x^2$ and inner radius $2 - x$. Hence the cross-sectional area is given by

$$A(x) = \pi \left((4 - x^2)^2 - (2 - x)^2\right).$$

To determine the limits of integration we equate the two radii, i.e.

$$4 - x^2 = 2 - x.$$

The above equation has the solution $x = -1$ and $x = 2$. Therefore the volume can now be calculated as

$$\begin{aligned} V &= \int_{-1}^{2} \pi \left((4 - x^2)^2 - (2 - x)^2\right) dx \\ &= \frac{108\pi}{5}. \end{aligned}$$