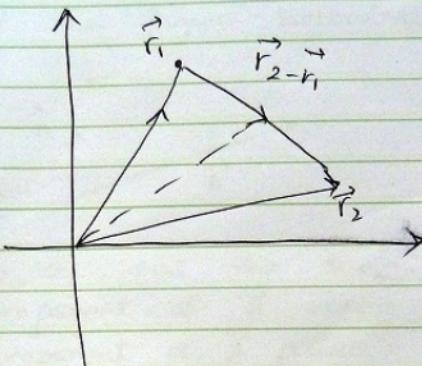


1-10

The two vectors \vec{r}_1 and \vec{r}_2 are shown in the figure below.



Since $r = |\vec{r}_2 - \vec{r}_1|$, and the vector $\vec{r}_2 - \vec{r}_1$ is a vector that joins \vec{r}_2 and \vec{r}_1 , a unit vector along the direction of line joining \vec{r}_2 and \vec{r}_1 is given by

$$\frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|} = \frac{\vec{r}_2 - \vec{r}_1}{r}$$

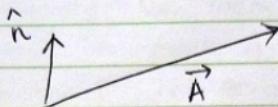
A vector from the origin till the point at distance xr from \vec{r}_1 along the line of \vec{r}_2 is hence given as

$$\vec{r}_1 + (xr) \left(\frac{\vec{r}_2 - \vec{r}_1}{r} \right) = (1-x)\vec{r}_1 + x\vec{r}_2$$

1.11 We need to show that

$$\vec{A} = (\vec{A} \cdot \hat{n})\hat{n} + (\hat{n} \times \vec{A}) \times \hat{n}$$

Consider the figure below

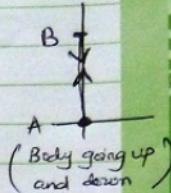
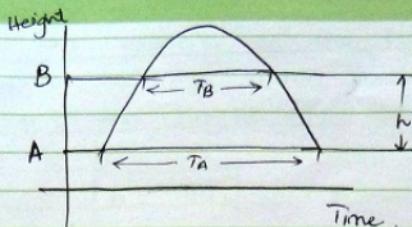


We can see that the term $(\vec{A} \cdot \hat{n})\hat{n}$ is the component of \vec{A} along the direction of \hat{n} . (Component of \vec{A} parallel to \hat{n}).

As for $(\hat{n} \times \vec{A}) \times \hat{n}$, notice that $\hat{n} \times \vec{A}$ is into the page by using the right-hand rule for cross-products. And $(\hat{n} \times \vec{A}) \times \hat{n}$ is giving us the "vertical" component of \vec{A} along the direction perpendicular to \hat{n} . Hence

$$\vec{A} = (\vec{A} \cdot \hat{n})\hat{n} + (\hat{n} \times \vec{A}) \times \hat{n}$$

1-12



Let the body be at point A at time $t=0$, with initial velocity v_A . Then the body goes up and comes down to point A in time T_A . Since the net displacement is zero, we can calculate T_A as

$$s = 0 = v_A T_A - \frac{1}{2} g T_A^2$$

$$\Rightarrow T_A = \frac{2v_A}{g} \quad \text{--- (i)}$$

At point B the velocity of the object is given by

$$v_B^2 - v_A^2 = 2(-g)h$$

$$\Rightarrow v_B = \sqrt{v_A^2 - 2gh} \quad \text{--- (ii)}$$

Since the net displacement from point B in time T_B is again zero, we have

$$s = 0 = v_B T_B - \frac{1}{2} g T_B^2 \quad \text{--- (iii)}$$

putting (i) and (ii) in (iii) and after simplification, we get

$$\frac{T_B^2}{T_A^2} = 1 - \frac{2gh}{v_A^2}$$

again using (i) in the above equation, we get

$$g = \frac{8h}{T_A^2 - T_B^2}$$

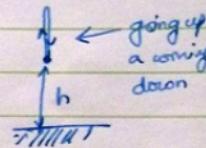
As required.

1.13 Let the velocity of the lift be v . Then the marble has the same initial velocity. At time $t = t_1$, the height of the marble and the lift is h .

Hence
$$h = vt_1$$

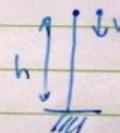
The marble initially goes up and then comes down to the same point where it was dropped from in time t_1 given by

$$\begin{aligned} s = 0 &= vt_1 - \frac{1}{2}gt_1^2 \\ \Rightarrow t_1 &= \frac{2v}{g} \quad \rightarrow (i) \end{aligned}$$



Then the marble falls through the height h under the influence of gravity but with initial speed v downwards. Time taken for this part would be t_2 and given as

$$s = vt_1 = vt_2 + \frac{1}{2}gt_2^2$$



$$\Rightarrow t_2 = -v \pm \sqrt{v^2 + \frac{4gvt_1}{2}} \quad \rightarrow (\ddot{u})$$

Now
$$t_1 + t_2 = T_2$$

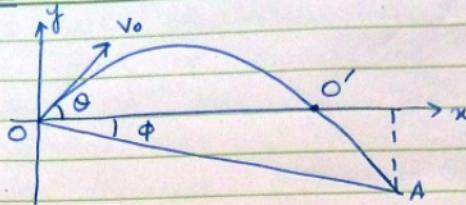
Adding (i) and (ii) and equating with T_2 , we get (after simplification)

$$V = \frac{g T_2^2}{2(T_1 + T_2)}$$

Since $h = VT_1$,

$$\Rightarrow h = \frac{g T_2^2 T_1}{2(T_1 + T_2)}$$

1.21



Equation for the mountain slope OA is given as

$$y = -(\tan \phi) x \quad \text{--- (i)}$$

The trajectory followed by the object is OO'A
Let the initial velocity be v_0 , then

$$x = v_0 \cos \theta t \quad \text{--- (ii)}$$

and

$$y = v_0 \sin \theta t - \frac{1}{2} g t^2 \quad \text{--- (iii)}$$

where x and y given above are the points along OO'A.

The body hits the mountain at point A. Therefore y is same at point A and hence we can equate (i) and (iii). This gives

$$-\tan \phi x = v_0 \sin \theta t - \frac{1}{2} g t^2 \quad \text{--- (iv)}$$

Now using (ii) to get

$$t = \frac{x}{v_0 \cos \theta}$$

and solving (iv) for x , we get

$$x = \frac{v_0^2 \sin 2\theta}{g} + \frac{2 v_0^2 \tan \phi \cos^2 \theta}{g}$$

The last equation gives the x-coordinate of the point of impact. The "range" or the distance OA will be maximised when x is maximised.

We therefore differentiate the last equation with respect to θ and put it equal to zero.

$$\frac{dx}{d\theta} = \frac{2\cos 2\theta V_0^2}{g} + \frac{2V_0^2(\tan \phi)(2\cos \theta)}{g}(-\sin \theta) = 0$$

$$\Rightarrow \cos 2\theta + 2\tan \phi (\cos \theta)(-\sin \theta) = 0.$$

$$\Rightarrow \cos 2\theta = \tan \phi \sin 2\theta$$

∴

$$\tan 2\theta = \frac{1}{\tan \phi}$$

$$\Rightarrow \boxed{\theta = \frac{1}{2} \tan^{-1} \left(\frac{1}{\tan \phi} \right)}$$

(We should evaluate $\frac{d^2x}{d\theta^2}$ at the above θ to check for maxima, this is left as an exercise).