

$$\ddot{x} = \ddot{y} \quad \text{---(i)} \quad T - M_2g = -M_2\ddot{y} \quad \text{---(ii)}$$

$$T = M_1\ddot{x} \quad \text{---(iii)}$$

Subtract (ii) from (iii)

$$M_2g = M_1\ddot{x} + M_2\ddot{y}$$

using (i)

$$M_2g = (M_1 + M_2)\ddot{x}$$

\Rightarrow

$$\ddot{x} = \frac{M_2}{M_1 + M_2} g.$$

\Rightarrow Integrate Twice

$$x = A + Bt + \frac{M_2}{M_1 + M_2} g \left(\frac{t^2}{2} \right)$$

where A and B are constants.

Since the system is at rest at $t=0$.

we have

$$x = 0 \text{ when } t = 0$$

and

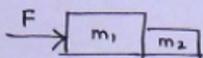
$$\dot{x} = 0 \text{ when } t = 0$$

This implies

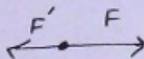
$$A = 0 \text{ and } B = 0.$$

So

$$x = \frac{M_2}{M_1 + M_2} g \left(\frac{t^2}{2} \right).$$

2.3

$$a = \frac{F}{m_1 + m_2} \quad (\text{acceleration of the system})$$

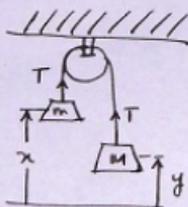
consider m_1 where F' is the force due to m_2

$$\text{then} \quad F - F' = m_1 a$$

$$F' = F - m_1 a$$

$$= F - \frac{F}{m_1 + m_2} \cdot m_1$$

$$F' = \frac{m_2}{m_1 + m_2} F$$

2.5

$$T - mg = m\ddot{x} \quad \text{and} \quad \ddot{x} = -\ddot{y} \quad (\text{if } m \text{ goes up } M \text{ goes down})$$

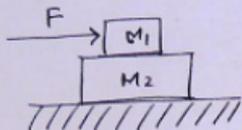
the same distance

$$T - Mg = M\ddot{y}$$

$$\Rightarrow (M - m)g = m\ddot{x} - M\ddot{y}$$

$$(M - m)g = -m\ddot{y} - M\ddot{y} \quad \Rightarrow \quad \ddot{y} = \left(\frac{m - M}{m + M} \right) g$$

2-7



In the case shown above, consider M_2

M_2 f_s (frictional force due to M_1/M_2 contact)
● →

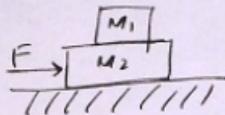
$$f_s = M_2 a. \quad \text{where } a = \frac{F}{M_1 + M_2}$$

and $f_s = \mu M_1 g.$

$$\Rightarrow \mu M_1 g = M_2 \frac{F}{M_1 + M_2}$$

$$\Rightarrow F = \mu \frac{M_1}{M_2} (M_1 + M_2)$$

Now consider



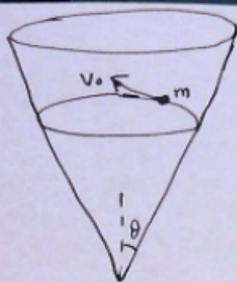
Consider forces on M_1

M_1 f_s
● →

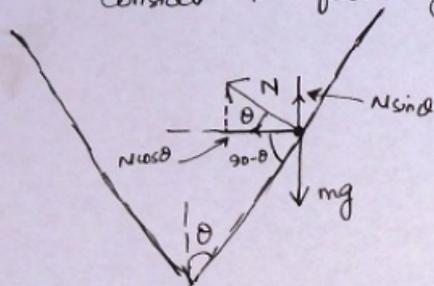
$$f_s = \mu M_1 g = M_1 a = M_1 \frac{F}{M_1 + M_2}$$

$$\Rightarrow \boxed{F = \mu (M_1 + M_2) g}$$

2.9



Consider the force diagram of m



$$N \sin \theta = mg \quad (\text{the mass is not moving vertically})$$

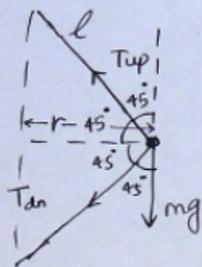
$$N \cos \theta = m \frac{v_0^2}{r} \quad \left(\frac{v_0^2}{r} \text{ is the radial acceleration} \right).$$

Dividing these two equations, we get

$$\tan \theta = \frac{mg}{m v_0^2 / r}$$

$$\Rightarrow \boxed{r = \frac{v_0^2 \tan \theta}{g}}$$

2-11



$$T_{up} \cos 45^\circ + T_{dn} \cos 45^\circ = m r \omega^2 \quad \text{where } r = l \cos 45^\circ$$

$$r = \frac{l}{\sqrt{2}}$$

$$\Rightarrow \frac{T_{up}}{\sqrt{2}} + \frac{T_{dn}}{\sqrt{2}} = \frac{m l}{\sqrt{2}} \omega^2$$

$$\Rightarrow T_{up} + T_{dn} = m l \omega^2 \quad \text{--- (i)}$$

And $T_{up} \sin 45^\circ = mg + T_{dn} \sin 45^\circ$

$$\Rightarrow T_{up} - T_{dn} = \sqrt{2} mg \quad \text{--- (ii)}$$

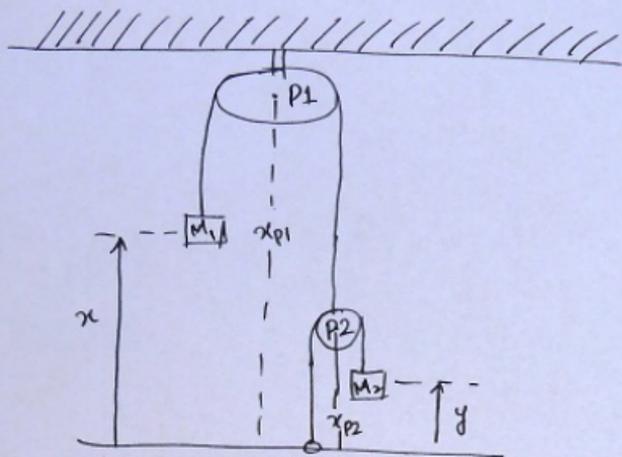
Add (i) & (ii)

$$\boxed{T_{up} = \frac{m}{2} (l \omega^2 + \sqrt{2} g)}$$

Similarly, subtract (ii) from (i)

$$\boxed{T_{dn} = \frac{m}{2} (l \omega^2 - \sqrt{2} g)}$$

2.13



Both strings (in P1 and P2) have constant lengths.

For Pulley 1

$$x_{p1} - x + \pi R + x_{p1} - x_{p2} = \text{constant}$$

Note that x_{p1} is constant but x_{p2} is not.

we ~~get~~ get

$$-\ddot{x} - \ddot{x}_{p2} = 0 \quad \text{or} \quad \boxed{\ddot{x} = -\ddot{x}_{p2}} \quad \text{---(i)}$$

For pulley 2

$$x_{p2} + \pi R + x_{p2} - y = \text{constant}$$

$$\Rightarrow \boxed{2\ddot{x}_{p2} = \ddot{y}} \quad \text{---(ii)}$$

using (i) and (ii) we get $\boxed{\ddot{y} = -2\ddot{x}} \quad \text{---(a)}$

Let the tension in the string of pulley 1 be T_1 ,
and the tension in the string of pulley 2 be T_2

Then

$$(b) - \boxed{T_1 - M_1 g = M_1 \ddot{x}}$$

$$(c) - \boxed{T_2 - M_2 g = M_2 \ddot{y}}$$

Force diagram for pulley 2 tells us

and

$$T_1 - 2T_2 = M_{p2} a_{p2}$$

mass of pulley 2
times acceleration
of pulley 2.

But pulley is considered
massless, so $M_{p2} = 0$.

and

$$(d) - \boxed{T_1 - 2T_2 = 0}$$

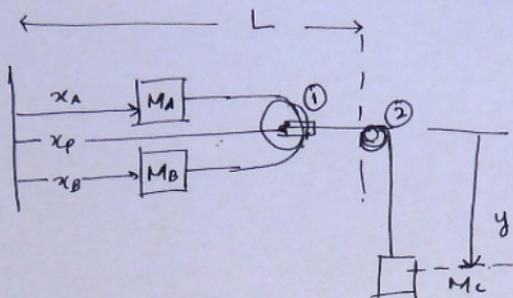
Solving (a), (b), (c), (d) simultaneously, we get

$$\begin{aligned} 2T_2 - M_1 g &= M_1 \ddot{x} \\ -2(T_2 - M_2 g) &= -2M_2 \ddot{x} \end{aligned}$$

$$2M_2 g - M_1 g = 4M_2 \ddot{x} + M_1 \ddot{x}$$

$$\Rightarrow \ddot{x} = \frac{2M_2 g - M_1 g}{4M_2 + M_1}$$

$$\Rightarrow \boxed{\ddot{x} = \frac{2m_2 - m_1}{4m_2 + m_1} g} , \boxed{\ddot{y} = \frac{-2(2m_2 - m_1)}{4m_2 + m_1} g}$$

2.14

Let L be the length of the table

Then, since the strings in each pulley are of constant length

$$(x_p - x_A) + (x_p - x_B) + 4R = \text{constant}$$

$$\Rightarrow \boxed{\ddot{x}_p = \frac{x_A + x_B}{2}}$$

Similarly $L - x_p + y = \text{constant}$

$$\boxed{\ddot{x}_p = \ddot{y}}$$

Let the tension in pulley (1) on the table be T_1

and let the tension in pulley (2) carrying M_C be T_2

Then for M_A and M_B

$$\boxed{T_1 = M_A \ddot{x}_A}$$

$$\boxed{T_1 = M_B \ddot{x}_B}$$

For pulley (1)

$$\boxed{T_2 - 2T_1 = 0}$$

(pulley is massless)

For M_c

$$M_c g - T_2 = M_c \ddot{y}$$

Juggling around with boxed equations, we get

$$\frac{\ddot{x}_A}{\ddot{x}_B} = \frac{M_B}{M_A}, \quad \ddot{y} = \frac{\ddot{x}_A + \ddot{x}_B}{2} \Rightarrow \ddot{y} = \ddot{x}_B + \frac{M_A}{M_B} \ddot{x}_A$$

$$\Rightarrow \ddot{x}_A = \frac{2\ddot{y}}{1 + \frac{M_A}{M_B}}$$

$$M_c g - 2T_1 = M_c \ddot{y}$$

$$M_c g - 2M_A \ddot{x}_A = M_c \ddot{y}$$

$$M_c g - 2M_A \left(\frac{2\ddot{y}}{1 + \frac{M_A}{M_B}} \right) = M_c \ddot{y}$$

$$\ddot{y} \left(M_c + \frac{4M_A M_B}{M_B + M_A} \right) = M_c g$$

\Rightarrow

$$\ddot{y} = g \left(\frac{M_c}{M_c + \frac{4M_A M_B}{M_B + M_A}} \right)$$

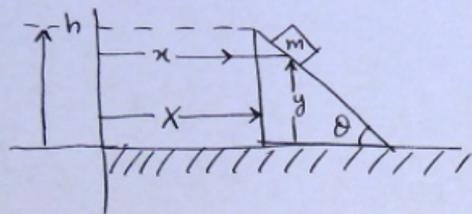
And

$$\ddot{x}_A = \frac{2g M_c M_B}{M_c (M_B + M_A) + 4M_A M_B}$$

And

$$\ddot{x}_B = \frac{2g M_c M_A}{M_c (M_B + M_A) + 4M_A M_B}$$

2.16



$$\frac{h-y}{x-x} = \tan \theta$$

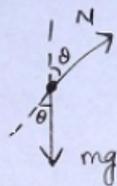
$$h-y = (x-x) \tan \theta$$

$$-\ddot{y} = (\ddot{x} - \ddot{X}) \tan \theta$$

but $\ddot{X} = A$ (given)

$$(i) \quad \boxed{-\ddot{y} = (\ddot{x} - A) \tan \theta}$$

Force diagram for m



$$\boxed{N \cos \theta - mg = m\ddot{y}}$$

$$\text{and } \boxed{N \sin \theta = m\ddot{x}}$$

Solving (i), (ii) and (iii) gives

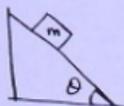
$$\boxed{\ddot{x} = \frac{A+g}{2}}$$

$$\boxed{\ddot{y} = \frac{A-g}{2}}$$

For
 $(\theta = 45^\circ)$

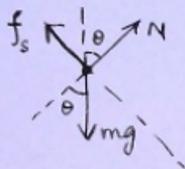
2.17

(d)



The frictional force always opposes motion.

The force diagram for m would be



At equilibrium

~~$mg \sin \theta = f_s$~~

and

$$\begin{aligned} mg \sin \theta &= f_s \\ mg \cos \theta &= N \end{aligned} \Rightarrow \tan \theta = \frac{f_s}{N} = \mu$$

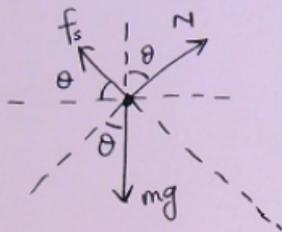
$$\Rightarrow \boxed{\mu = \tan \theta}$$

(b) We want to find a μ which keeps the block stationary and does not allow it to slip "down".

In this case force of friction acts upward along the inclined surface.

The problem is similar to 2.16, with the addition exception of frictional force f_s

The force diagram of m would be



(i) — $N \sin \theta - f_s \cos \theta = ma$ (Horizontal forces)

(ii) — $N \cos \theta + f_s \sin \theta = mg$ (Vertical forces, no vertical acceleration).

use

$f_s = \mu N$ and divide (i) and (ii)

$$\frac{a}{g} = \frac{N \sin \theta - \mu N \cos \theta}{N \cos \theta + \mu N \sin \theta}$$

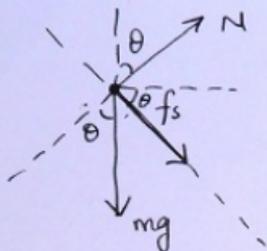
$$a = \frac{g (\sin \theta - \mu \cos \theta)}{(\cos \theta + \mu \sin \theta)}$$

or

$$\boxed{a_{\min} = \frac{g (\tan \theta - \mu)}{(1 + \mu \tan \theta)}}$$

(c) Now if we accelerate the ~~block~~ wedge too much the block would tend to move up along the wedge. The frictional force in this case would act "down" along the ~~the~~ inclined plane.

The force diagram now become



$$N \sin \theta + f_s \cos \theta = ma$$

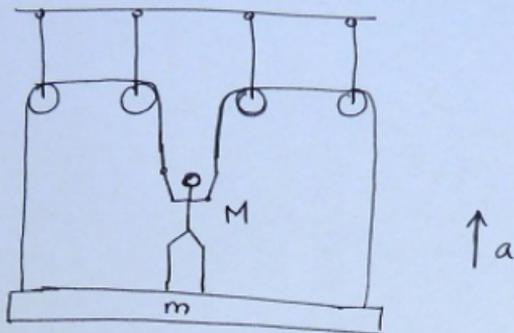
$$N \cos \theta - f_s \sin \theta = mg$$

- Again using $f_s = \mu N$ and dividing the above two equations, we get

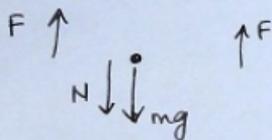
$$\frac{a}{g} = \frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta}$$

or

$$a_{\max} = \frac{g(\tan \theta + \mu)}{(1 - \mu \tan \theta)}$$



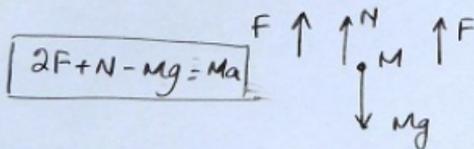
Force diagram ~~for~~ ^{for} m



$$\boxed{2F - N - mg = ma}$$

(where N is the force exerted by painter on the platform)

Force diagram for M



$$\boxed{2F + N - Mg = Ma}$$

Adding the boxed equations, we get

$$4F - (m+M)g = (m+M)a$$

\Rightarrow

$$\boxed{a = \frac{4F - (m+M)g}{m+M}}$$