

Let  $a$  be the acceleration of the system,

then

$$\frac{F}{M_1 + M_2 + M_3} = a \quad \text{--- (i)}$$

Let  $T$  be the tension in the string joining

$M_2$  and  $M_3$ . Then

$$T = M_2 a \quad \text{and} \quad T = M_3 g$$

(For  $M_2$ ) (For  $M_3$ )

Dividing these equations we get

$\Rightarrow$

$$\frac{M_2 a}{M_3 g} = 1 \quad \text{or} \quad a = \frac{M_3}{M_2} g$$

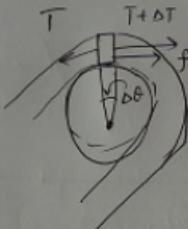
$\Rightarrow$  Using this in (i)

$$\frac{M_3}{M_2} g = \frac{F}{M_1 + M_2 + M_3}$$

$$\Rightarrow \boxed{F = \frac{M_3}{M_2} g (M_1 + M_2 + M_3)}$$

2.24

Let the tension at a point be  $T$  and a point closer to the point  $B$  by an angle  $\Delta\theta$  be  $T + \Delta T$ . Since load at  $A$  is much larger, it wants to pull the string down and friction is in the opposite direction.



$$\text{We have } T + \Delta T + f = T$$

$$\Rightarrow \Delta T = -f.$$

but  $f = \mu N$  and  $N = \Delta \theta T$  (see solved example 2-13 at page 90 of the book)

$$\Rightarrow \Delta T = -\mu \Delta \theta T$$

$$\Rightarrow \frac{1}{T} \Delta T = -\mu \Delta \theta$$

or

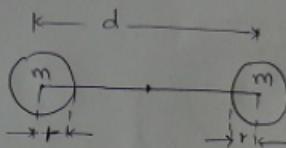
$$\frac{dT}{T} = -\mu d\theta$$

$$\Rightarrow \ln T = -\mu \theta + k,$$

$$T = k e^{-\mu \theta}$$

$$\text{At } \theta = 0, T = T_A$$

$$\Rightarrow T = T_A e^{-\mu \theta}$$



Let the mass of each sphere be  $m$ , and the distance b/w their centres be  $d$ . Let the radius of each sphere be  $r$ .

The gravity  $\alpha$  supplies the radial acceleration and we have

$$m\left(\frac{d}{2}\right)\omega^2 = \frac{G(m)(m)}{(d/2)^2}$$

but  $\omega = \frac{2\pi}{T}$ , and rearranging, we get

$$\frac{(2\pi)^2 \cdot \left(\frac{d}{2}\right)^3}{Gm} = T^2$$

$$\Rightarrow T = \frac{\pi d \sqrt{d}}{\sqrt{2Gm}}$$

So  $T$  decreases as  $d$  decreases and the minimum  $d$  can be  $2r$ , when the spheres are just touching each other. So

$$T_{\min} = \frac{\pi 2r \sqrt{r}}{\sqrt{Gm}}$$

2.26



At a distance  $r$  from the centre of earth only the spherical mass inside the radius  $r$  pulls on the object.

Mass inside sphere of radius  $r$  = volume  $\times$  density

$$= \frac{4}{3}\pi r^3 \cdot \rho$$

Force due to gravitation

$$F = -\frac{G\left(\frac{4}{3}\pi r^3 \rho\right)m}{r^2}$$

$$\Rightarrow m\ddot{r} + G\frac{4}{3}\pi r \rho m = 0$$

$$\Rightarrow \boxed{\ddot{r} + \left(\frac{4}{3}G\pi\rho\right)r = 0}$$

Since  $\frac{4}{3}G\pi\rho > 0$ , the above equation  
is of the form

$$\ddot{r} + \omega^2 r = 0 \Rightarrow \omega = \sqrt{\frac{4}{3}G\rho}$$

So the time period will be

$$T = \frac{2\pi}{\omega} = \frac{2\pi\sqrt{3}}{2\sqrt{\pi G\rho}} = \frac{\sqrt{3\pi}}{\sqrt{G\rho}}$$

For a satellite in low orbit,  
the centripetal force is provided by gravitation.

Hence

$$mR\omega^2 = \frac{GmM}{R^2}$$

where  $m$  is the mass of the satellite,  $M$  is the mass of the earth, and  $R$  is the radius of the earth.  
 $M$  can be written as  $M = \frac{4}{3}\pi R^3 \rho$ , where  $\rho$  is the density of the earth. Hence, we get

$$mR\omega^2 = \frac{Gm(\frac{4}{3}\pi R^3 \rho)}{R^2}$$

$$\Rightarrow \omega^2 = \frac{4}{3}\pi \rho G$$

or

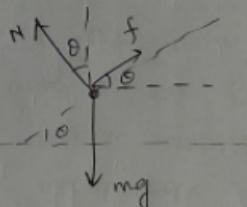
$$\omega = \sqrt{\frac{4}{3}\pi \rho G}$$

which is same as before, and gives

$$T = \frac{2\pi}{\omega} = \frac{\sqrt{3\pi}}{\sqrt{\rho G}} \text{ as before.}$$

2.28

The force diagram would be



There are 3 forces on the car. The force of friction is upwards if the speed is slow and the car is tending to slide down.

For this case, we have

$$N \cos \theta + f \sin \theta = mg \quad (\text{vertical components})$$

and

$$N \sin \theta - f \cos \theta = \frac{mv^2}{R} \quad (\text{horizontal components})$$

Since  $f = \mu N$ , dividing both equations we get

$$\frac{\sin \theta - \mu \cos \theta}{\cos \theta + \mu \sin \theta} = \frac{v^2}{Rg}$$

$$\Rightarrow v_{\min} = \sqrt{\left( \frac{\tan \theta - \mu}{1 + \mu \tan \theta} \right) Rg}$$

For the maximum speed, the direction of friction is reversed, so we get

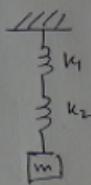
$$N \cos \theta - f \sin \theta = mg$$

$$N \sin \theta + f \cos \theta = \frac{mv^2}{R}$$

$$\Rightarrow v_{\max} = \sqrt{\left( \frac{\tan \theta + \mu}{1 - \mu \tan \theta} \right) Rg}$$

2.31

(a)



Let the total displacement in m be x where

$$x = x_1 + x_2$$

$x_1$  — displacement in spring 1

$x_2$  — — x — v — v — 2

The stretching force F is same on each spring

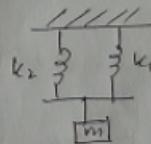
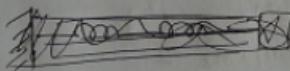
So

$$x = \frac{F}{k_1} + \frac{F}{k_2} = F \left( \frac{k_1 + k_2}{k_1 k_2} \right) = \frac{F}{k_{\text{eff}}} \Rightarrow k_{\text{eff}} = \frac{k_1 k_2}{k_1 + k_2}$$

where  $k_{\text{eff}}$  is the combined effective spring constant of the system.

$$\Rightarrow \omega = \sqrt{\frac{k_{\text{eff}}}{m}} = \sqrt{\frac{k_1 k_2}{m(k_1 + k_2)}}$$

(b)



In this case the net force on m is

$F = -k_2 x - k_4 x$ , where x is the displacement in each spring (which is same for each spring).

∴  $F = -(k_2 + k_4)x$

So  $k_{\text{eff}} = k_2 + k_4$  and  $\omega = \sqrt{\frac{k_2 + k_4}{m}}$



There is no radial force on  $m$ , hence

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0$$

$\Rightarrow$

$$\ddot{r} - r\omega^2 = 0$$

we can easily check that this differential equation has the solution

$$r = A e^{wt} + B e^{-wt}$$

(because  $\ddot{r} = A\omega^2 e^{wt} + B\omega^2 e^{-wt}$ )  
and  $\ddot{r} - r\omega^2 = 0.$

So

$$r = \omega$$

If  $A \neq 0$  then as  $t \rightarrow \infty$ ,  $r \rightarrow \infty$ .

Let's see if we can make  $A$  zero.

$$\text{at } t=0, \quad r(0) = A + B.$$

and

$$\dot{r}(0) = Aw - Bw.$$

Solving for  $A$ ,

$$A = \frac{\omega r(0) + \dot{r}(0)}{2w}$$

Equating this with zero

$$\Rightarrow \omega r(0) + \dot{r}(0) = 0$$

$$\Rightarrow \dot{r}(0) = -\omega r(0)$$

If this condition is met  
then  $r(t)$  decreased  
as time increases.

2.34

Since the radius  $r$  decreases due to the string being pulled with const velocity  $V$ . we have,

$$r = r_0 - Vt \Rightarrow \dot{r} = -V, \ddot{r} = 0$$

Using polar coordinates, we know that

$$\dot{a}_r = \ddot{r} - r\omega^2$$

$$\dot{a}_r = -(r_0 - Vt)\omega^2$$

and

$$a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta} = 2\dot{r}\omega + r\dot{\omega}$$

$a_\theta = 0$  because there is no force in  $\hat{\theta}$  direction.

So we have

$$(r_0 - Vt)\dot{\omega} + (-2V)\omega = 0.$$

$$\Rightarrow \frac{\dot{\omega}}{\omega} = \frac{2V}{r_0 - Vt} \quad (\text{integrate this})$$

$$\Rightarrow \ln(\omega) = 2V \frac{\ln(r_0 - Vt)}{-V} + K_1$$

$$\omega = k / (r_0 - Vt)^2$$

Since  $\omega = \omega_0$  where  $t=0$ , we have

$$\omega_0 = k / r_0^2 \Rightarrow k = \omega_0 r_0^2$$

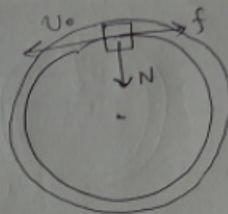
$\Rightarrow$

$$\boxed{\omega = \frac{\omega_0 r_0^2}{(r_0 - Vt)^2}}$$

For the force on the string, we simply note

that  $T = m a_r = -m(r_0 - Vt)\omega^2$  (minus shows the

$$\boxed{T = -m\omega_0 r_0^2 (r_0 - Vt)^{-1}}$$
 Tension is inward)



The only force along  $\hat{\theta}$  direction is due to  $f$ . In fact  $f$  acts along  $-\hat{\theta}$  direction.

We have

$$m a_{\theta} = -f = -\mu N = \mu m \frac{v^2}{l} \quad (i)$$

where  $N$  is the normal reaction and  $N = \frac{mv^2}{l}$ .

Since this is the only force that causes circular motion.

$$\text{Also } a_{\theta} = 2\dot{r}\hat{\theta} + r\ddot{\theta} \quad (ii)$$

In this case,  $\dot{r} = 0$  because  $r = l$  is a constant.

So compare (i) and (ii)

$$m l \ddot{\theta} = -\mu m \frac{v^2}{l}$$

$$\Rightarrow l\ddot{\theta} = -\mu \frac{v^2}{l}$$

but since  $v = l\dot{\theta} \Rightarrow \dot{v} = l\ddot{\theta}$ , so

$$\dot{v} = -\mu \frac{v^2}{l}$$

or

$$\frac{\dot{v}}{v^2} = -\mu/l$$

Integrating

$$-v^{-1} = -\mu/l t + k,$$

$$\Rightarrow v = \frac{1}{\mu/l t + k}$$

Now, we know  $v = v_0$  when  $t = 0$ .

$$\therefore v_0 = \frac{1}{0+k} \Rightarrow k = \frac{1}{v_0}$$

$$\therefore v = \frac{1}{\mu/l t + \frac{1}{v_0}}$$

$$\boxed{v = \frac{v_0}{[(\mu v_0/l)t + 1]}}$$