Exploiting Interference - The Michelson Interferometer

PHY 300 - Junior Phyics Laboratory

Hassan Bukhari
Roll no: 2012-10-0152
Department of Physcis LUMS SSE

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1 Introduction

What was once a simple proof of the wave theory of light, interference, is now used for many practical applications in physics. The Michelson Interferometer is a simple setup in which the path difference between the interfering waves can be directly controlled (as opposed to the double slit experiment, where it depends on the spacing between the slits). In this experiment we exploited the experimental setup of the Michelson Interferometer to obtain the wavelength of a He-Neon Laser and the refractive index of a thin glass slide. The uncertainties involved in calculating both of these variables are also discussed in detail.

2 Theoretical Background

2.1 The Michelson Interferometer

The Michelson interferometer was assembled as shown in Figure 1. The beam splitter (BS) evenly splits the beam coming from the laser and sends it in the direction of Mirror 1 (M1) and Mirror 2 (M2). M2 is mounted on a movable track whose movement can be controlled by a computer controlled servo motor (Model: TDC001). Lens 1 (L1) is a plano-convex lens with focal length 35mm, while Lens 2 (L2) is also plano-convex but has focal length 25.4mm and spreads the fringes onto the screen so that they can be observed better.

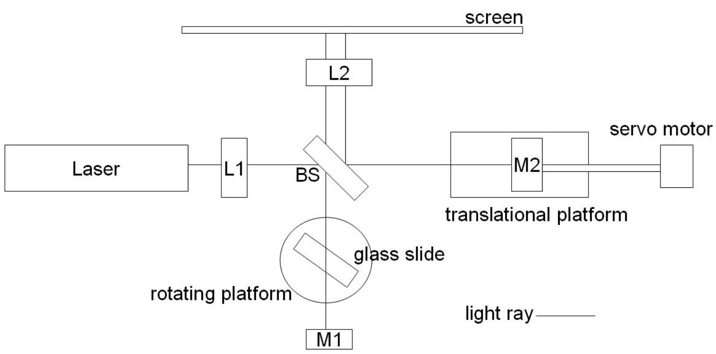

Figure 1: Experimental Setup Diagram

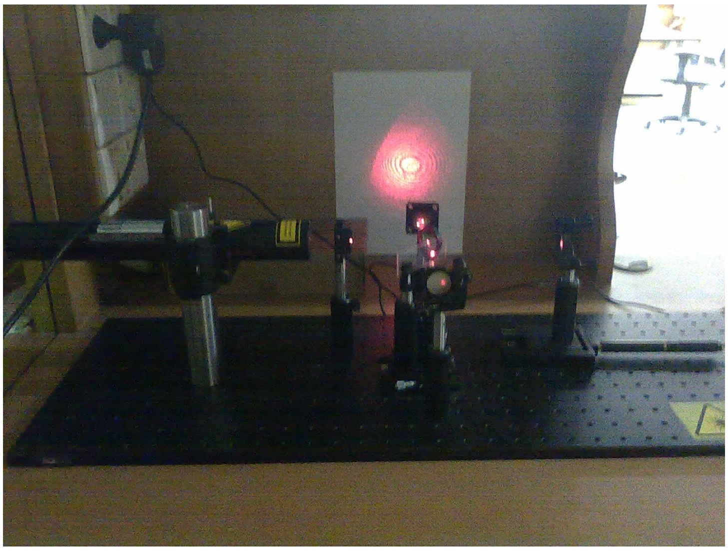

Figure 2: Experimental Setup

3 Experimentation

3.1 Calculating the Wavelength

The servo motor was moved using its computer interface. The number of fringes that were displaced was counted and this process repeated 6 times to reduce experimental error. The results are shown in Table 1. Ch Pos gives the current position of M2 and Cu Pos gives the position it was directed to go to. The number of fringes displaced were observed. The first reading was taken by using the graph mode of the interface but then we switched to the numerical display as controlling the motor was more convenient.

After the Beam splitter splits the beam the two components travel different path lengths and the once coherrent waves are now seperated by a phase and they create an interference pattern on the screen. When we move M2 the path difference changes due to a change in path length. The change in path difference by moving M2 is shown in Equation 1 and its relation to fringe displacement (N) is given in Equation 1.

$$\delta path = 2 \times \text{distance moved by M2} = N\lambda$$

$$\lambda = \frac{2 \times \text{distance moved by M2}}{N}$$

Table 1: Fringe displacement by moving M2

Reading No	Ch Pos/mm	Cu Pos/mm	Distance moved by M2/μm	Fringes Displaced (N)	λ/nm
1	0.2740	0.289	10	30	666.67
2	0.2857	0.2957	10	31	645.16
3	0.3100	0.3200	10	30	666.67
4	0.3300	0.3500	20	60	666.67
5	0.3500	0.3600	10	31	645.16
6	0.507	0.517	10	31	645.16

The mean value of λ from the 6 readings is 655 nm.

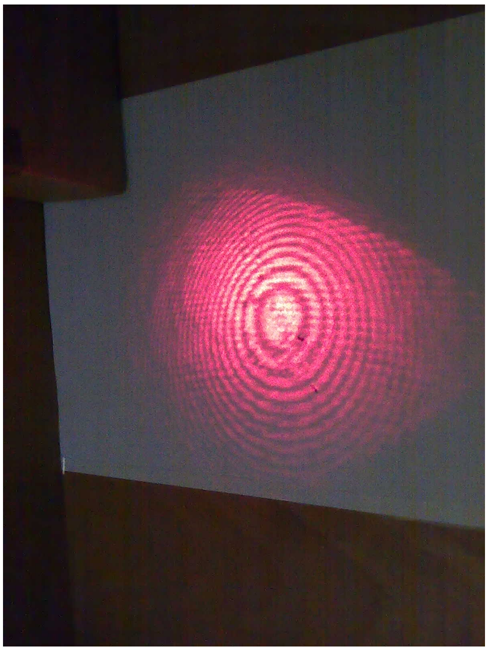

Figure 3: Interference Fringes

3.2 Uncertainty in λ

3.2.1 Type A

This is due to the random error introduced by non-uniform experimental technique and conditions. Standard Deviation = 11.8 nm. Standard Error = $\frac{S.D.}{\sqrt(6-1)} = 4.8 \simeq 5nm$

3.2.2 Type B

This is due to the limit of resolution of the equipement. The position of M2 was calculated to minimum resolution of 100nm by the computer interface. Hence standard error = $frac100nmsqrt(12) = 28.9nm$

3.2.3 Combined Uncertainty

The combined uncertainty in λ is:

$$u = \sqrt(5^2 + 28.9^2) = 29nm$$

Therefore $\lambda = 655 \pm 29$nm This value of the uncertainty puts the manufacturer specification value of the He-Neon of 633nm within the error bars of our measured value.

4 Calculating the Refractive Index

4.1 Theoretical Background

A thin glass slide was inserted into a rotating apparatus between the BS and mirror 1. Mirror 2 was kept stationary at some arbitrary distance. The presence of the glass slide introduces a phase difference in the light passing through it dependant on the length of the path that it travels through it and the refractive index. The path of the light through the glass slide is shown in Figure 2. When the glass slide was rotated the path length of the light beam through the glass slide was increased. Althought the change in path length was minute even after rotating through 35 degrees, the change in path length could be counted in units of lambda i.e. by counting the fringe displacement caused by the rotation.

The phase change of the beam from going from point A to B is

$$\frac{2\pi nAB}{\lambda} = \frac{2\pi nd}{\lambda cos(\theta)}$$

Here n is the refractive index of the medium, λ is the wavelength of the light and θ is the angle from the horizontal. θ is both the incident angle and the angle that we measure during rotation. The horizontal here is paralell to M1. Where as the beam in the other arm passes through air. We multiply by two as the beam passes through the glass twice. This idea is matured to give Equation 1.

$$ng = \frac{(2t - N\lambda)(1 - cos(\theta))}{2t(1 - cos(\theta)) - N\lambda} \quad (1)$$

We rotated the glass slide in segments of 5 degrees and noted the number of fringe displacements. The slide was rotated up to 35 degrees from the horizontal. The number of fringes displaced become too large to count accurately beyond this level. We rotated the slide very slowly by attaching two meter rules and then rotating it by moving the very tip. However, even this setup did not allow for readings beyond 35 degrees with accuracy. This procedure was repeated 5 times and the results are summarized in Table 2. Calculating the refractive index required the width (t) of the glass plate. This was obtained using a micrometer screw gauge. 5 readings were taken and are given in Table 3.

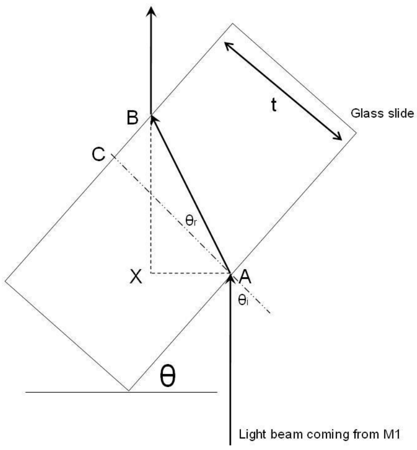

Figure 4: Experimental Setup

Table 2: Width of the glass slide

Thickness (t)/ mm
1.37
1.37
1.37
1.37
1.37

The gauge had a zero error of -0.10 mm and so t = 1.47mm.

4.2 Theoretical vs. Actual Results

Equation 1 can be rearranged to give a relationship between N and θ. This is shown in Equation 2.

$$N = \frac{2t(ng - ngcos(\theta) - 1 + cos(\theta))}{\lambda(n - 1 - cos(\theta))} \quad (2)$$

Table 3: Fringe displacement by rotating Glass Slide

Rotation/theta	N1	N2	N3	N4	N5	Nmean	n (refractive index)
0-5	6	6	7	7	6	6.4	1.59
0-10	19	20	20	20	20	19.8	1.402
0-15	41	43	43	42	41	42.0	1.366
0-20	73	74	72	72	75	73.2	1.347
0-25	115	118	113	114	114	114.8	1.340
0-30	169	169	159	164	156	163.4	1.32
0-35	236	229	211	225	216	223.2	1.31

This is plotted in Figure 5 for various N. The mean value of our results from the 5 runs have also been plotted onto this graph. The mean value of the refractive index is 1.382 $\approx$ 1.4 with Standard Deviation of 0.97

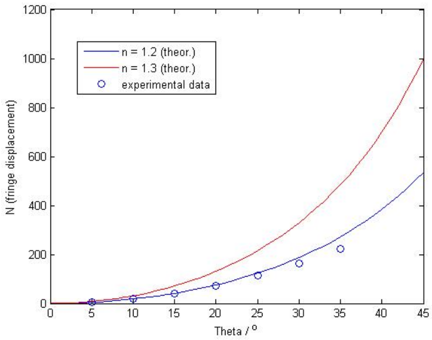

Figure 5: Experimental Result vs. Theoretical Prediction

4.2.1 Going Further

Studies have used the Michelson Interferometer to calculate the width of very thin transparent materials using the same technique that we used to calculate the refractive index. We took a cover slip from the brownian motion experiment and tried to calculated its thickness using our interferometer. θ was varied from 0 to 40. Fringe displacement was noted to be less than that of the previous slide. The experiment was repeated thrice. Results are summarized in Table 4.

Only 3-4 fringes were displaced when we rotated 0-10 degrees. This results in larger fractional error. Fractional error is discussed further in the next section. Even 0-40 degree rotation resulted in a fringe shift of only 43-46 fringes.

Equation 1 can futher be manipulated to give Equation 3 which gives a formula for the width (t) in terms of the refractive index, N and θ. For our calculation n was taken to be 1.4 and λ to be 655 nm as calculated earlier.

$$t = \frac{nN\lambda - N\lambda(1 - cos(\theta))}{2n(1 - cos\theta) + 2cos(\theta) - 2} \tag{3}$$

Table 4: Fringe displacement by rotating Cover Slip

Theta	N1	N2	N3	Mean N	Width (t)/mm)
10	3	2	3	2.67	0.199
20	11	11	10	10.67	0.194
30	24	25	23	24.0	0.186
40	46	44	43	44.33	0.181

Mean value of t = 0.187 $\approx$ 0.19 mm. This corresponds poorly with the value calculated by the micrometer of

4.3 Uncertainty in the Refractive Index

There is a seperate uncertainty associated with measuring n at different values of θ.

4.3.1 Type A

To calculate the refractive index we had to measure fringe displacement (N), the width (t) of the glass plate and λ as was calculated in the previous part. Fringes could be missed while in the 30-35 degree range as they moved very fast. However, this error is hard to quantify and can only be reduced with repeated readings. The width readings were all the same and so we believe there is no type A uncertainty related to it. Error in λ has been established earlier. S.D N for θ 0-5 = 0.547

4.3.2 Type B

The lowest resolution for measuring θ was 1 degree. The micrometer screw gauge had a lowest count of 0.01 mm and also had a zero error of -0.10 mm which was corrected for in the readings.

4.3.3 Combined Uncertainty

The total uncertainty in N is

The total uncertainty in θ is

The total uncertainty in t is The combined uncertainty in n is given by

$$u^2(n) = u_N^2(n) + u_\theta^2(n) + u_t^2(n) + u_\lambda^2(n)$$

5 Conclusion